# Conjectures regarding the nonlinear geometry of visual neurons

## Supplementary Information

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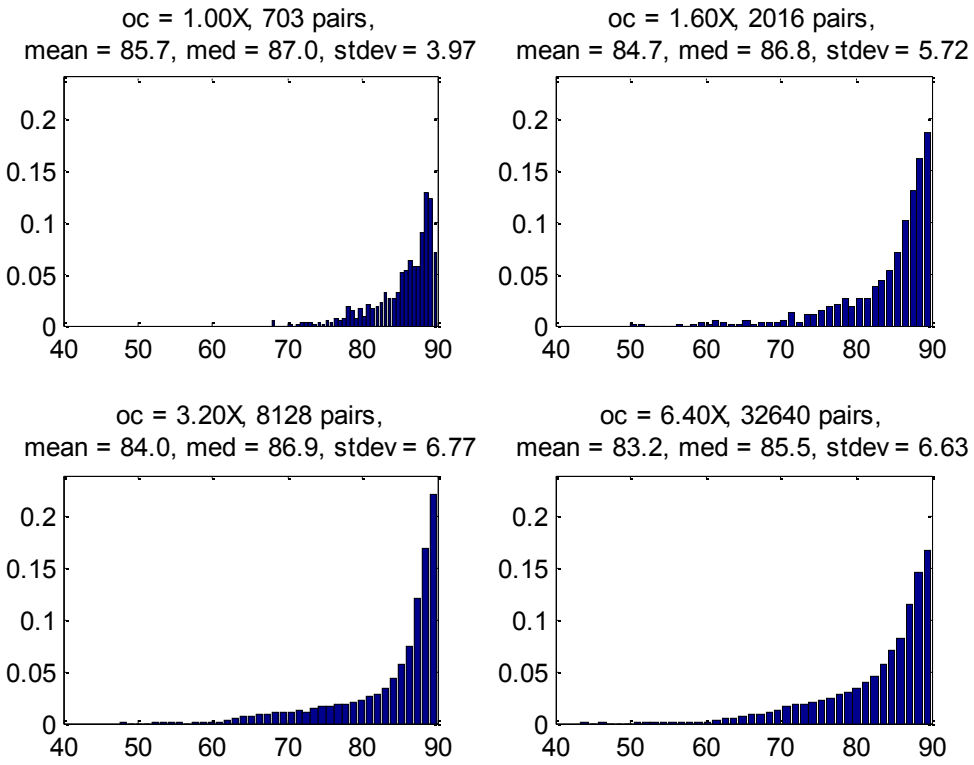


Fig S1: The histograms representing the angles between every pairs of basis vectors for 8x8 sparse coding networks that are a) 1X overcomplete, b) 1.6X overcomplete, c) 3.2X overcomplete and d) 6.4X overcomplete. The mean angle between pairs for every network is approximately 90 degrees, but the standard deviation for the angle grows with the degree of overcompleteness.

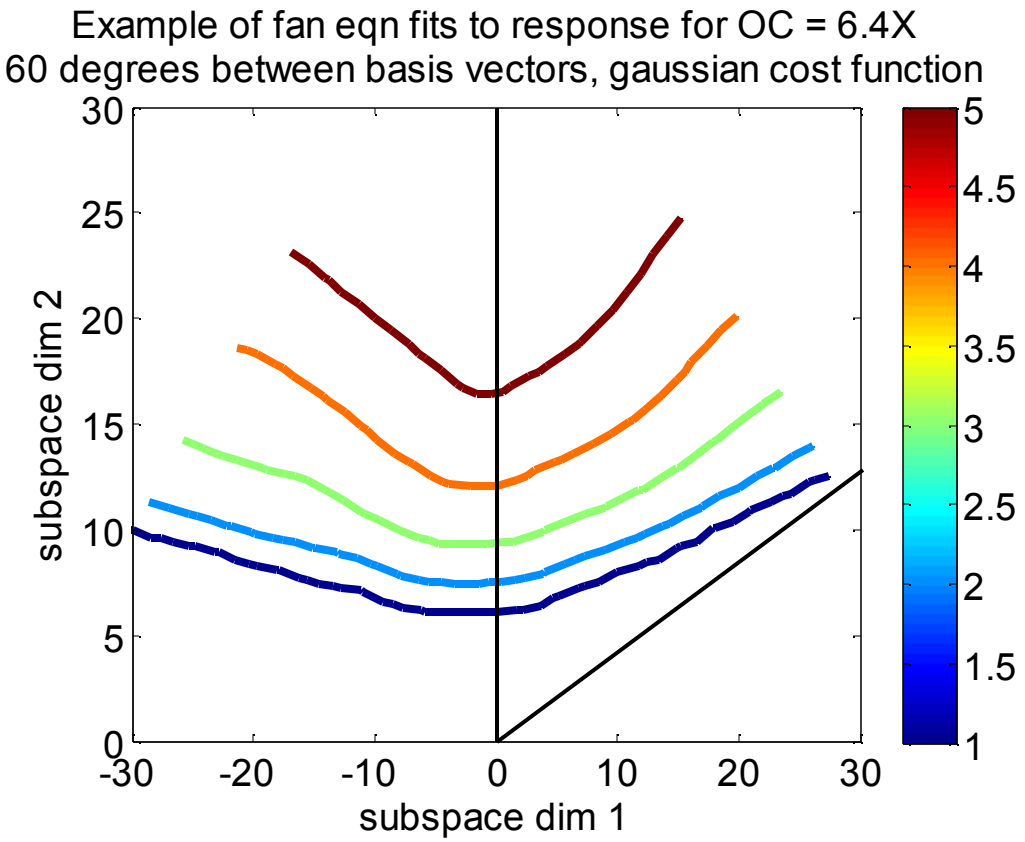


Fig. S2a: The iso-response contours at responses of 1, 2, 3, 4 and 5 for a neuron from a 6.4X overcomplete sparse coding network using a **Gaussian cost function** in a 2D subspace determined by this neuron's basis vector and its closest neighbor. The basis vector points in the direction (0,1), while the neighboring basis vector points at (2,1), as shown in black. Note the curvature of the iso-response contours away from the neighboring basis vector.

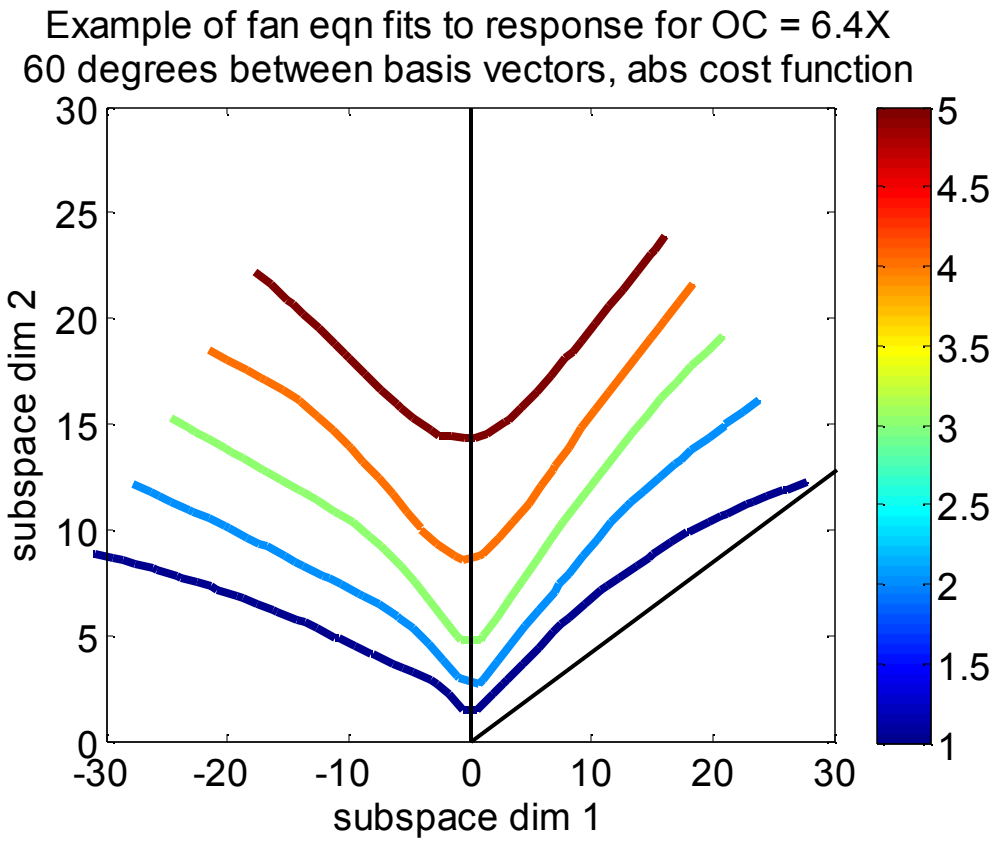


Fig. S2b: The iso-response contours at responses of 1, 2, 3, 4 and 5 for a neuron from a 6.4X overcomplete sparse coding network using an **absolute value cost function** in a 2D subspace determined by this neuron's basis vector and its closest neighbor. The basis vector points in the direction (0,1), while the neighboring basis vector points at (2,1), as shown in black. Note the curvature of the iso-response contours away from the neighboring basis vector.

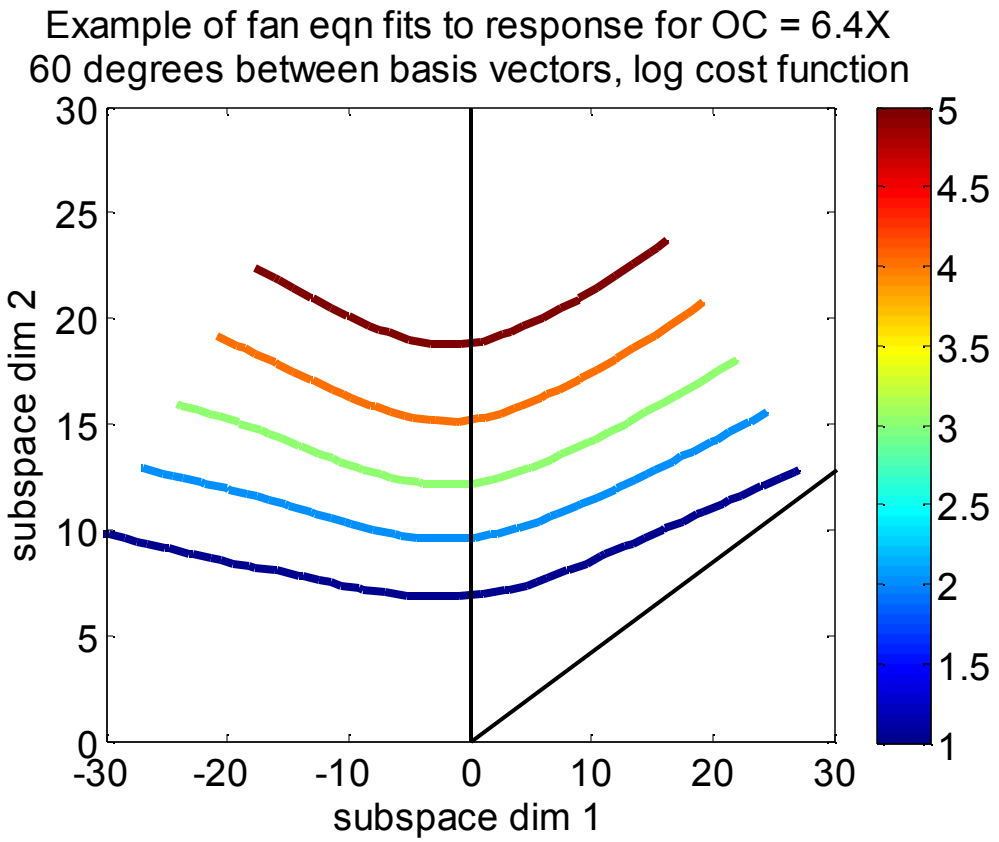


Fig. S2c: The iso-response contours at responses of 1, 2, 3, 4 and 5 for a neuron from a 6.4X overcomplete sparse coding network using a **log cost function** in a 2D subspace determined by this neuron's basis vector and its closest neighbor. The basis vector points in the direction (0,1), while the neighboring basis vector points at (2,1), as shown in black. Note the curvature of the iso-response contours away from the neighboring basis vector.

Isocontours with Parabolic Fit y = a*x²+b, Gaussian cost function

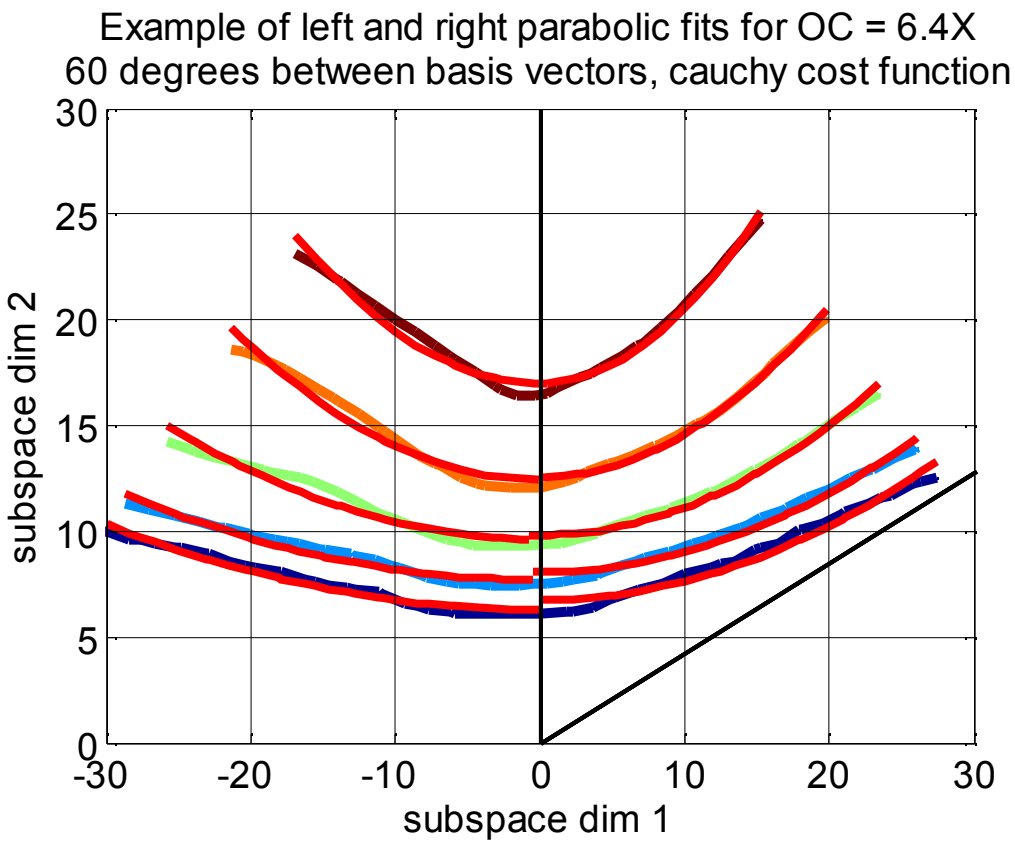


Fig. S3a: The iso-response contours from the above neuron fit with a parabola (y = a*x²+b) for the **Gaussian cost function**. The red lines indicate the parabolic fit; the left and right branches of the isocontours are fit independently. Note that each fit results in different *a* parameters for each iso-response level.

Isocontours with Parabolic Fit y = a*$x^2$+b, absolute value cost function

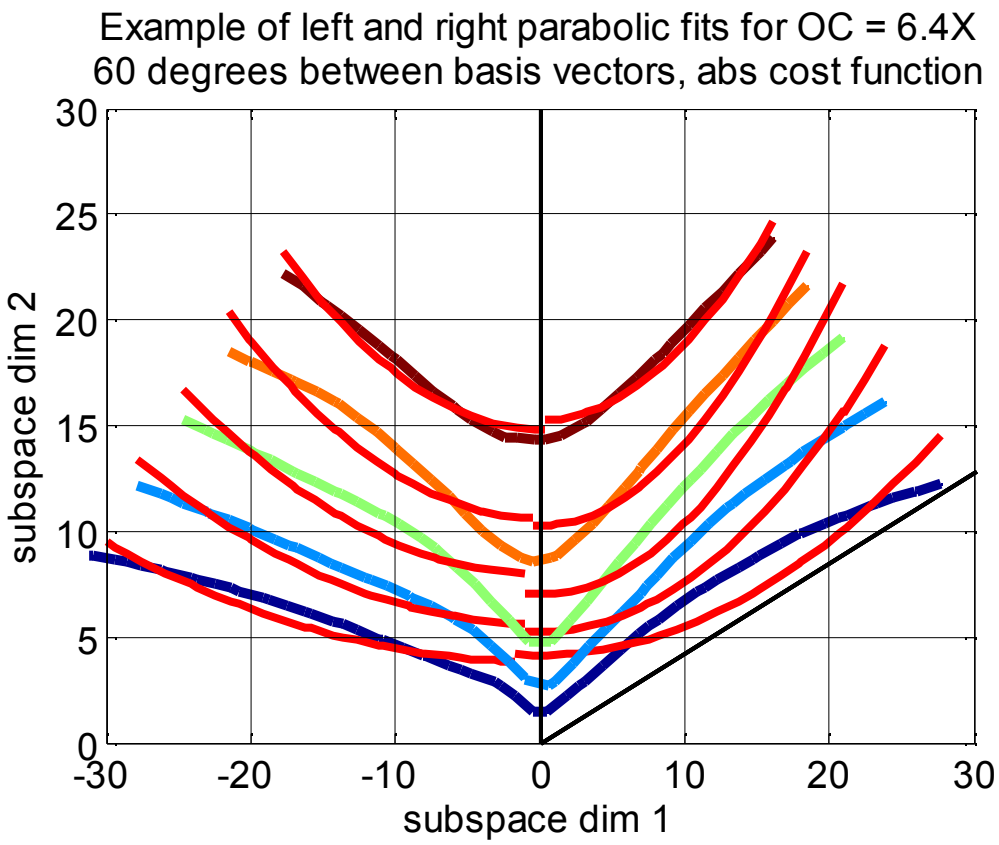


Fig. S3b: The iso-response contours from the above neuron fit with a parabola (y = a*$x^2$+b) for the **absolute value cost function**. The red lines indicate the parabolic fit; the left and right branches of the isocontours are fit independently. Note that each fit results in different *a* parameters for each iso-response level.

Isocontours with Parabolic Fit $y = a*x^2+b$, log cost function

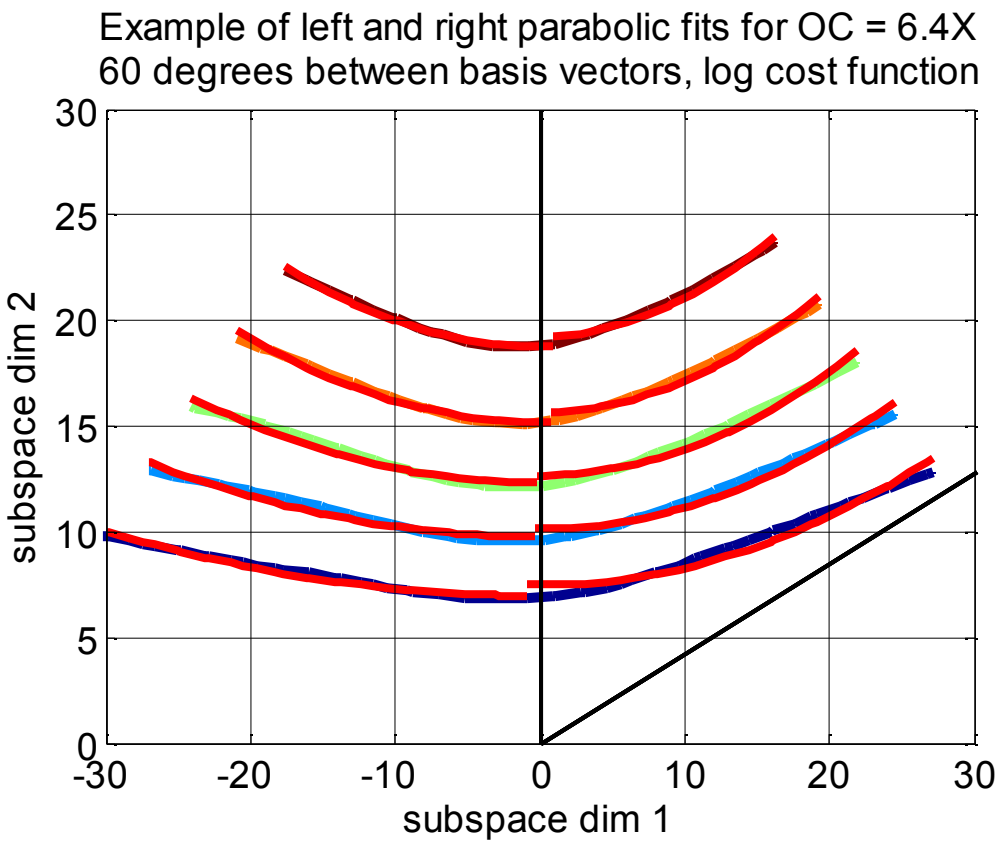


Fig. S3c: The iso-response contours from the above neuron fit with a parabola ($y = a*x^2+b$) for the **log cost function**. The red lines indicate the parabolic fit; the left and right branches of the isocontours are fit independently. Note that each fit results in different *a* parameters for each iso-response level.

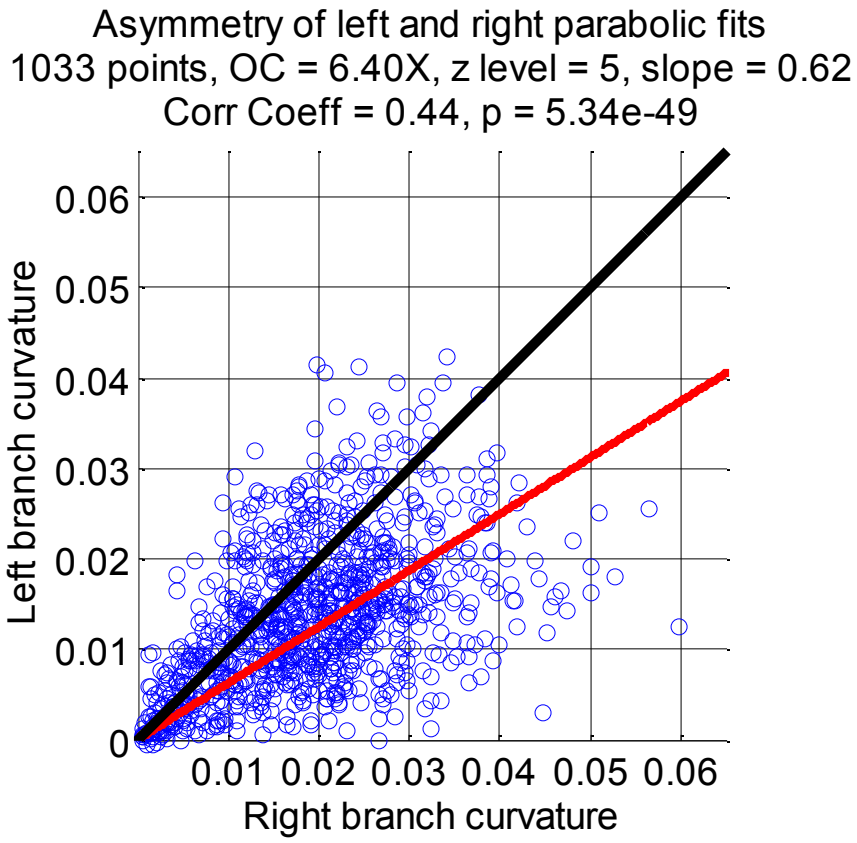


Fig. S4: Note that the fits are often asymmetric. This plot shows the correlation between the curvature (parabolic fit parameter *a*) of the left and right branches for responses in 2D subspaces from the 6.4X overcomplete network. Note that the right branch (or the branch closer to the neighboring basis vector) often shows greater curvature, as evidenced by the linear fit slope of 0.62, shown by the red line. The black line has a slope of 1, and points would fall along this if the fits were perfectly symmetric.

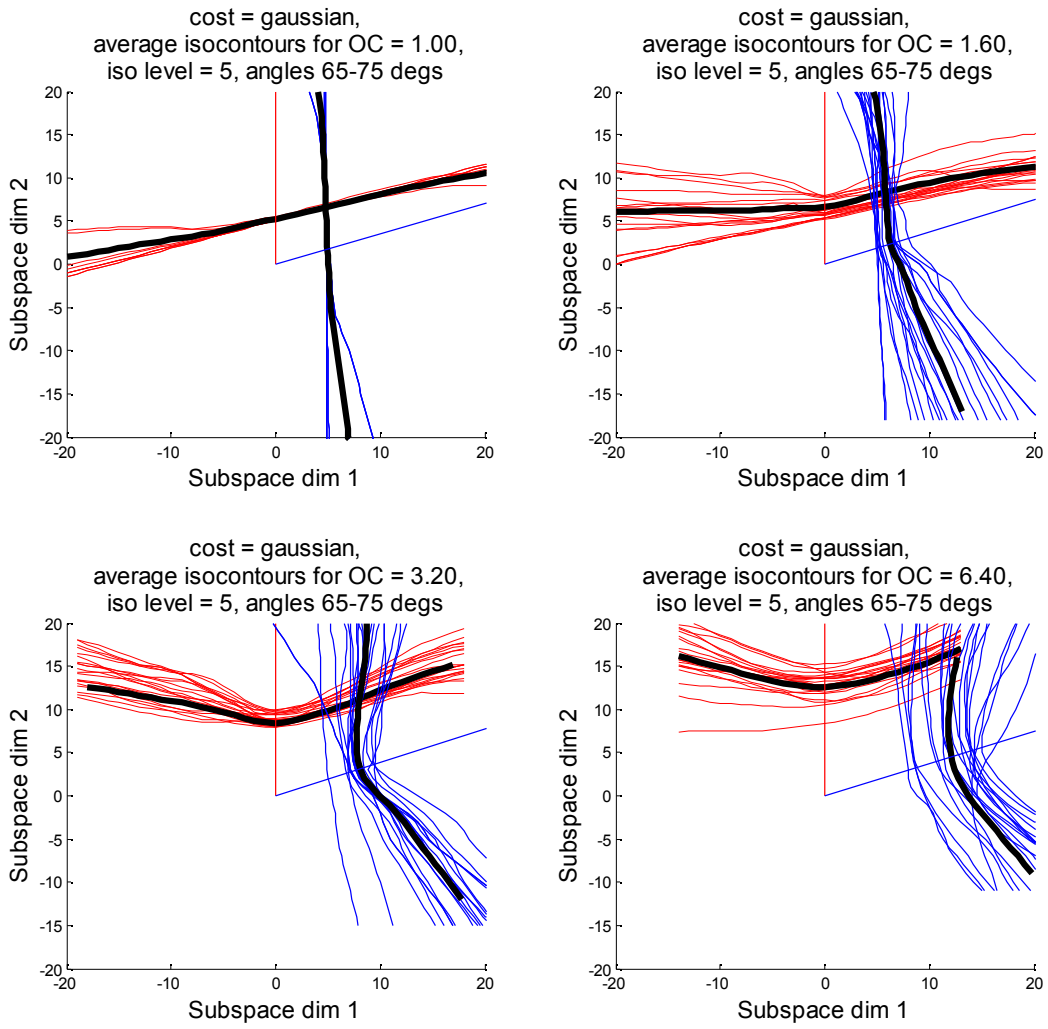


Fig. S5a-d: **Gaussian cost function**, four levels of overcompleteness (top left: 1X/critically sampled, top right: 1.6X overcomplete, bottom left: 3.2X oc, bottom right: 6.4X oc), 20 random isocontours (blue and red) and the average isocontours (black) for neurons with basis vectors at 65 deg – 75 deg. The basis vectors are shown in red and blue in the directions of (0, 1) and (2, 1).

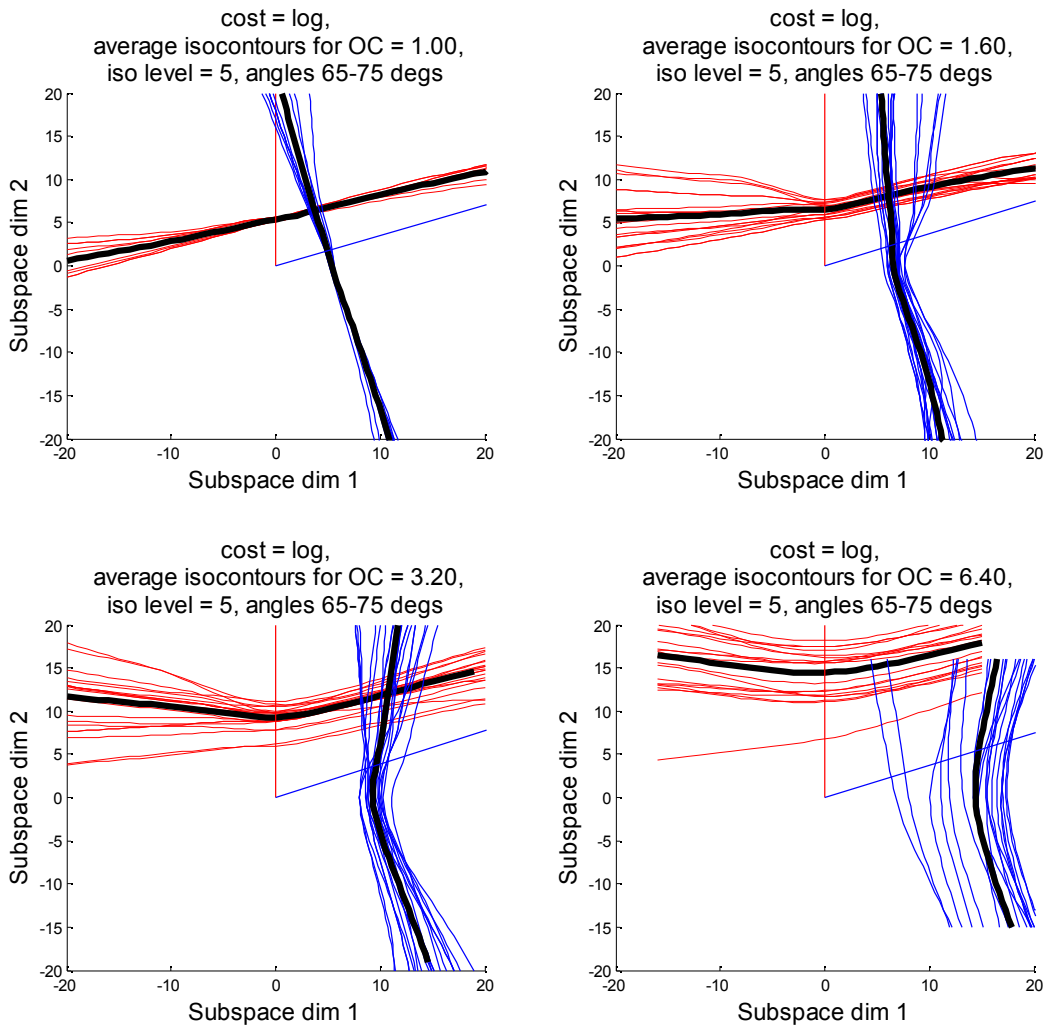


Fig. S5e-h**: Absolute value cost function**, four levels of overcompleteness (top left: 1X/critically sampled, top right: 1.6X overcomplete, bottom left: 3.2X oc, bottom right: 6.4X oc), 20 random isocontours (blue and red) and the average isocontours (black) for neurons with basis vectors at 65 deg – 75 deg. The basis vectors are shown in red and blue in the directions of (0, 1) and (2, 1).

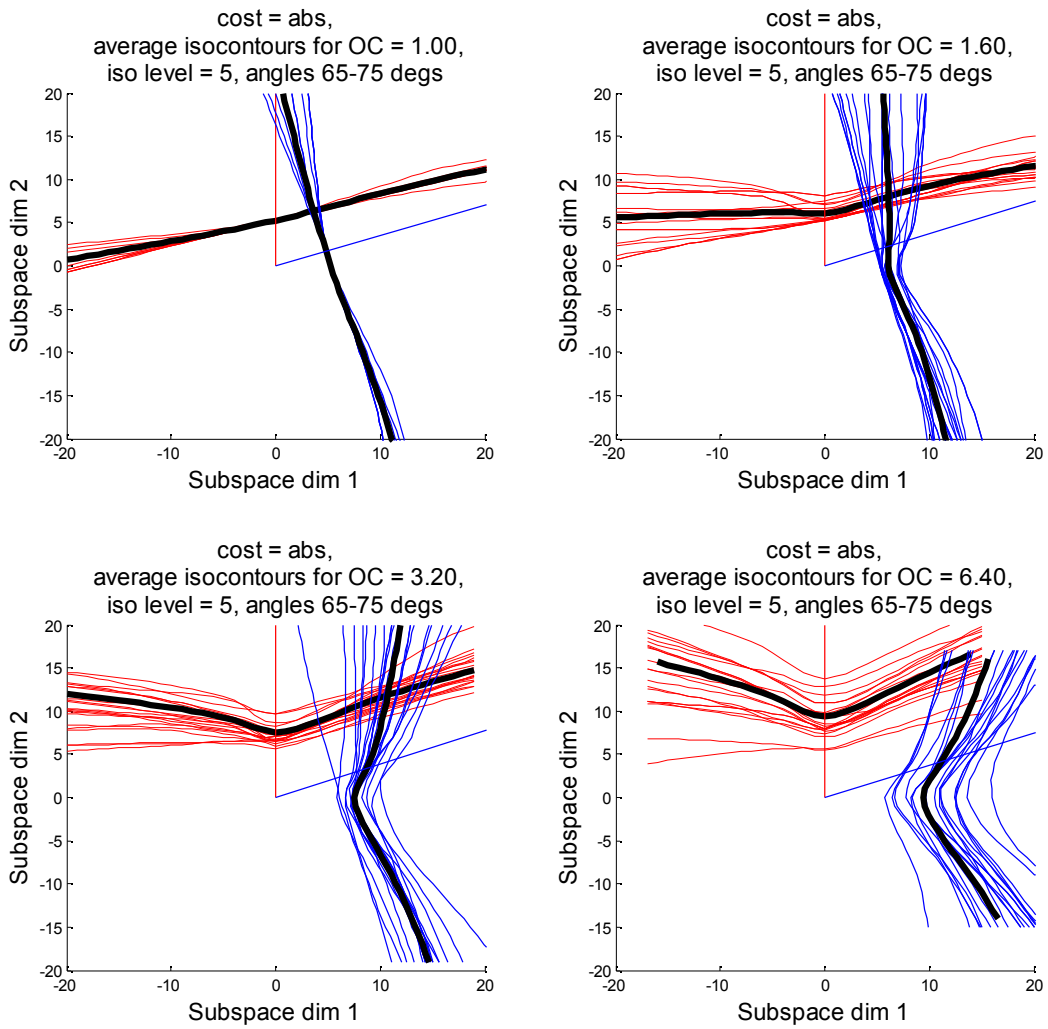


Fig. S5i-l: **Log cost function**, four levels of overcompleteness (top left: 1X/critically sampled, top right: 1.6X overcomplete, bottom left: 3.2X oc, bottom right: 6.4X oc), 20 random isocontours (blue and red) and the average isocontours (black) for neurons with basis vectors at 65 deg – 75 deg. The basis vectors are shown in red and blue in the directions of (0, 1) and (2, 1).

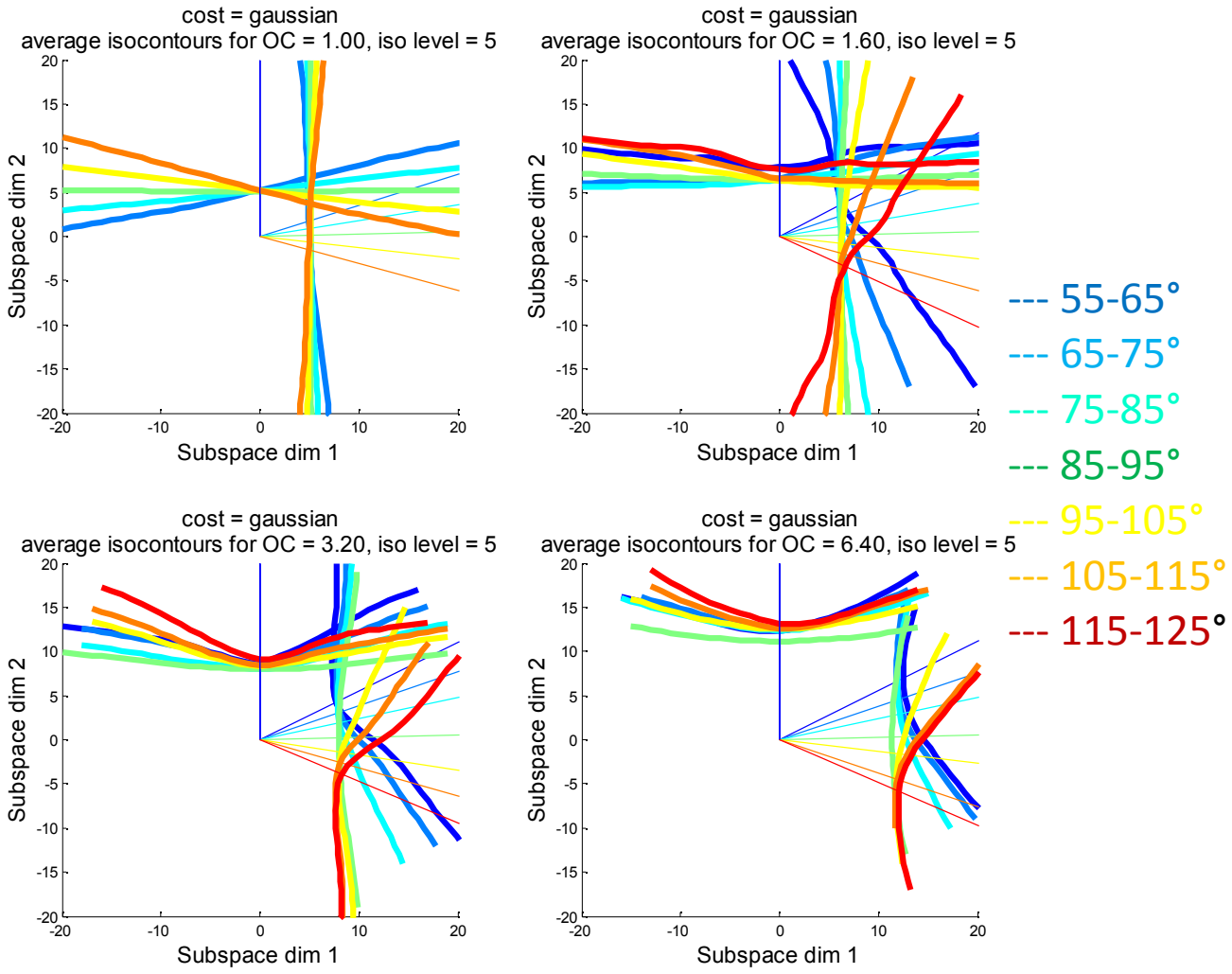


Fig. S6a-d**: Gaussian cost function**, average iso-contours (black lines from Fig. S8, but shown in color here) for neurons with basis vectors at a number of ranges of degrees: 55-65 degs (blue), 65-75 degs (light blue), 75-85 degs (cyan), 85-95 degs (green), 95-105 (yellow), 105-115 (orange), 115-125 degs (red).

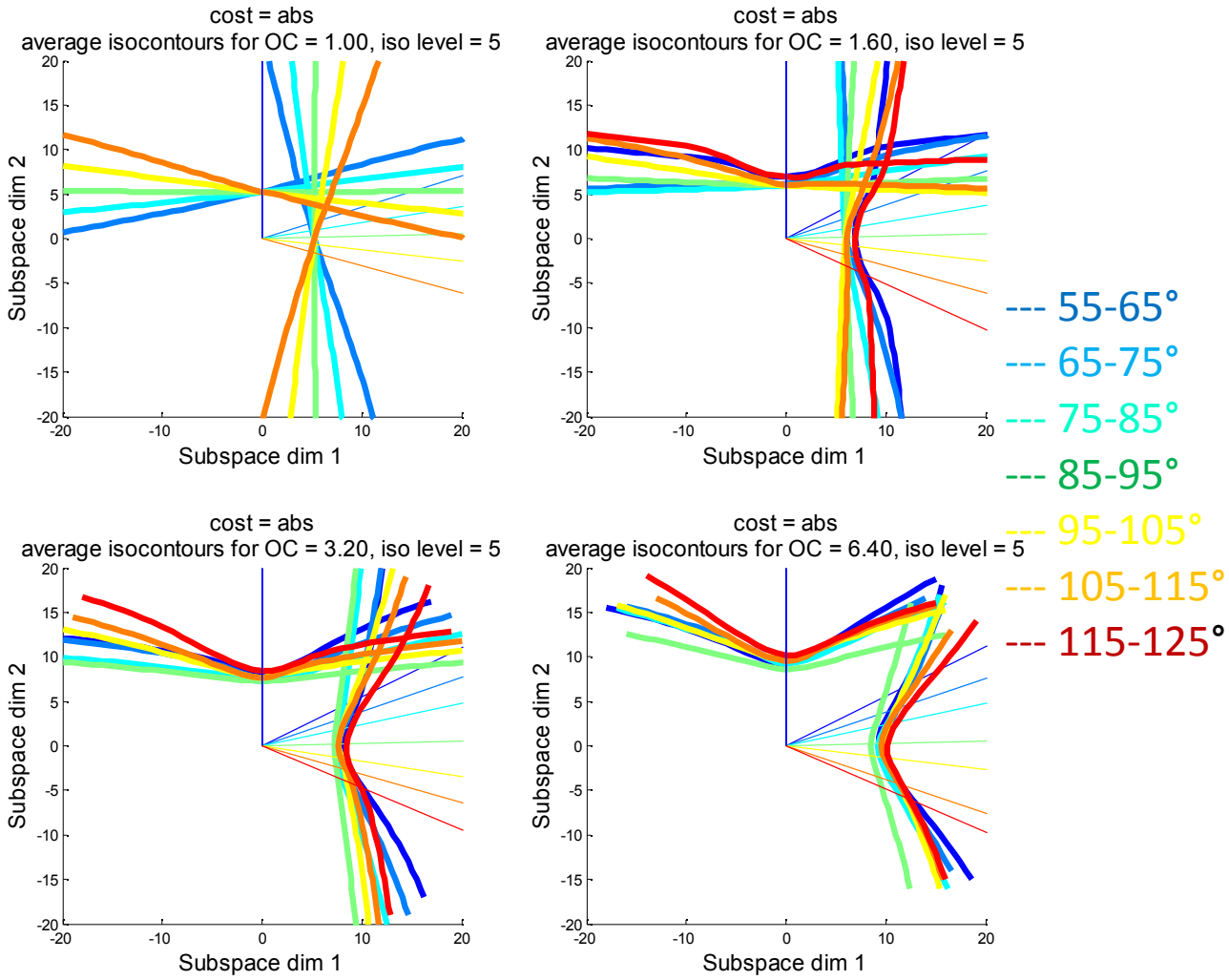


Fig. S6e-h: **Absolute value cost function**, average iso-contours (black lines from Fig. S8, but shown in color here) for neurons with basis vectors at a number of ranges of degrees: at 55-65 degs (blue), 65-75 degs (light blue), 75-85 degs (cyan), 85-95 degs (green), 95-105 (yellow), 105-115 (orange), 115-125 degs (red).

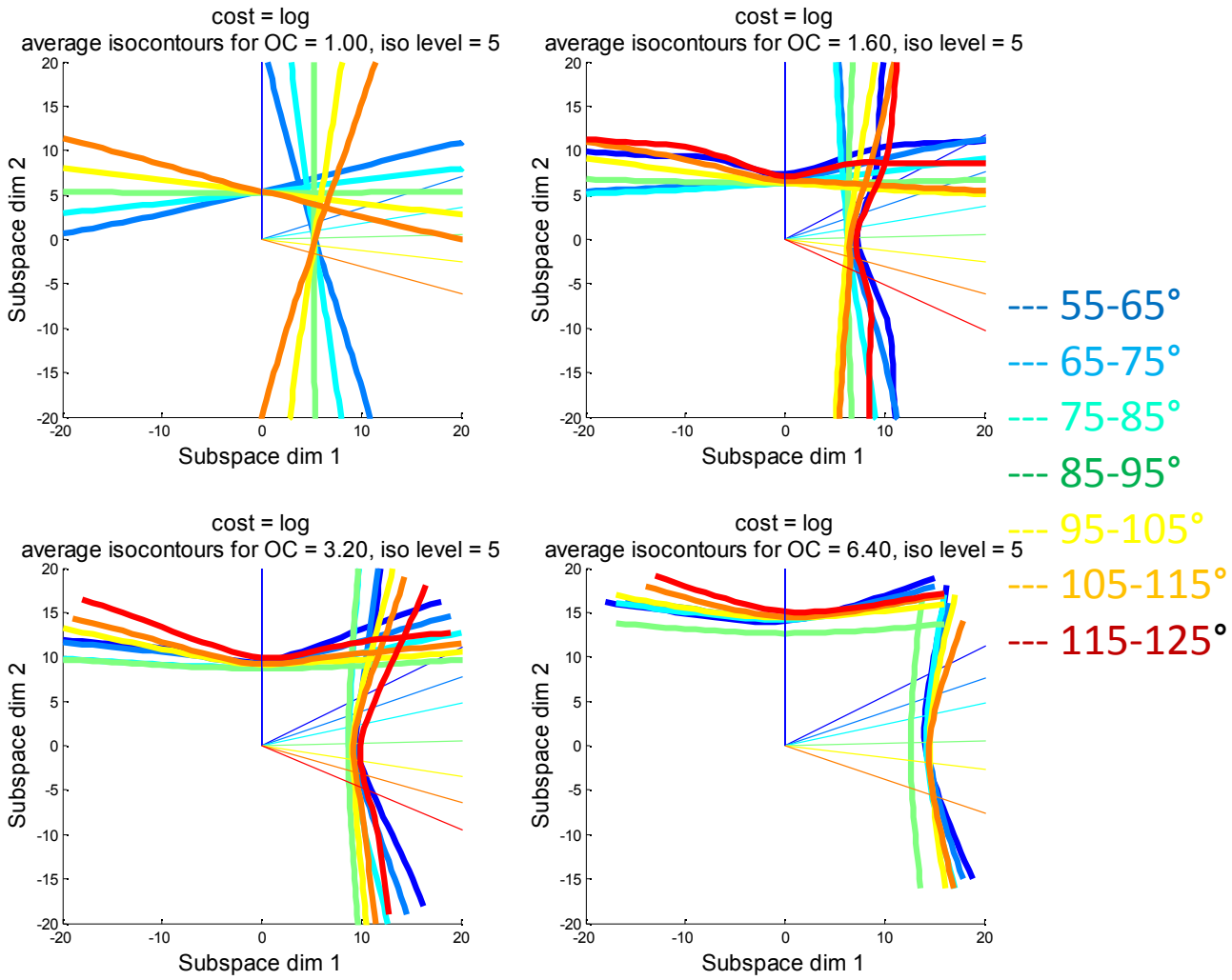


Fig. S6i-l: **Log cost function**, average iso-contours (black lines from Fig. S8, but shown in color here) for neurons with basis vectors at a number of ranges of degrees: 55-65 degs (blue), 65-75 degs (light blue), 75-85 degs (cyan), 85-95 degs (green), 95-105 (yellow), 105-115 (orange), 115-125 degs (red).